GAUSSIAN-BEAM MODELING OF ULTRASONIC TRANSDUCERS USING NEAR-FIELD EXPERIMENTAL DATA

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INTRODUCTION

We wish to develop a model for the pitch-catch response of real transducers which is both accurate and efficient. Wen and Breazeale [1, 2] have shown that the fields of a uniformly active, planar disk transducer can be modelled by a small number of coaxial Gaussian beams (eg, 10 or 15). Margetan, Thompson and Gray [3] have similarly modelled the fields of a uniformly active, planar disk transducer in terms of Hermite Gaussian beams, and further picked the radius of the ideal transducer to match the main lobe of experimental data collected with a very small receiver,to approximate a point probe. Here we use the expansion of Wen and Breazeale and reciprocity to model the pitch-catch response of two transducers facing each other and having parallel axes, as a function of the displacement vector between the two transducers. Further, we fit this pitch-catch model directly to experimental pitch-catch data by choosing the parameters of the Gaussian beams, without assuming that the transducers are uniformly active, planar disks. Finally we show that the model developed by fitting over one set of displacements accurately describes the experiments over a disjoint set.

THEORY

The expansion of Wen and Breazeale for the velocity gradient ϕ of a transducer centered at the origin and facing along the z axis is

$$\phi(r,z) = \sum_{n=1}^{N} \frac{A_n}{(1+2iB_n z/k)} \exp\left[-\frac{B_n r^2}{(a^2+2iB_n z/k)} -ikz \right], \quad (1)$$

where r is the radial displacement from the axis, $k=\omega/v$ is the wavenumber at frequency ω in a medium with acoustic velocity v, and A_n and B_n are the complex parameters of the N Gaussian beams in the expansion. a is an arbitrary distance introduced to make B_n nondimensional, and is taken as the nominal radius of the transducer. A_n gives the complex amplitude of each beam. Each beam has a narrowest point. B_n controls the width at the narrowest point and where it occurs along z. This expansion is an exact solution of the parabolic wave equation, which is an approximation to the wave equation itself. The paraxial expression of reciprocity [4] for the pitch-catch received voltage from a transmitter which generates ϕ_1 and a receiver which would generate ϕ_2 is

$$v(x,y,z) = \eta \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \, \phi_1(x',y',z) \, \phi_2(x'-x,y'-y,0) \tag{2}$$

where η is a normalization factor accounting for driving signal, etc., x and y are the Cartesian coordinates perpendicular to z, and the integrals are taken over a closed surface surrounding one of the transducers, typically a plane separating them and closed at infinity. For the pitch-catch geometry shown in Fig. 1, the pitch-catch voltage for the two transducers, from Eqs. (1) and (2) is

$$v(r,z) = e^{ikz} \sum_{m=1}^{N} A_m \exp\left[-B_m\left(\frac{r}{a}\right)^2\right] \sum_{n=1}^{N} A'_n(z) \exp\left[\frac{B_m^2(r/a)^2}{B'_n(z)+B_m}\right] \frac{\pi}{B'_n(z)+B_m} \tag{3}$$

where r is the radial displacement between the transducers, z the axial displacement, η has been absorbed into A_n and

$$A'_n(z) = \frac{A_n}{1+i\frac{B_n z}{\pi z_R}}, \tag{4}$$

$$B'_n(z) = \frac{B_n}{1+i\frac{B_n z}{\pi z_R}}, \tag{5}$$

$$z_R = a^2/\lambda . \tag{6}$$

Eq. 3 is the theoretical result used below to fit and predict experimental data.

EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 2. Alignment of the transducers is a critical step in the procedure. We assumed that the axes of the translation stage were perpendicular to one another. We placed a block of material with two very flat and parallel sides between the two transducers. The moveable transducer was operated in pulse-echo mode, and scanned over the block in x and y, and the position of the block adjusted to insure that its two flat and parallel surfaces were parallel to the x and y axes of the translation stages. The orientation of the two transducers was then adjusted to maximize the pulse-echo response of each. Finally, the block was removed, and the x-y origin located at the maximum of the pitch-catch response.

At each position (ie, displacement vector) of interest a waveform was digitized with 20 MHz sampling. Fig. 3 shows an example. The direct pitch-catch pulse occurs rougly between 2 and 5 μs in this example. Multiples occur at about 11 and 20 μs. Fig. 4 shows the spectrum of only the direct pulse, confirming that the transducers have a peak in response near 2.25 MHz, and that the 20 MHz sampling rate is adequate. Each waveform

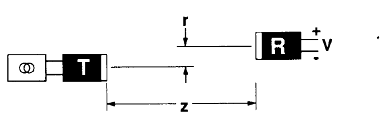

Fig. 1 Geometry for pulse-echo model and experiments.

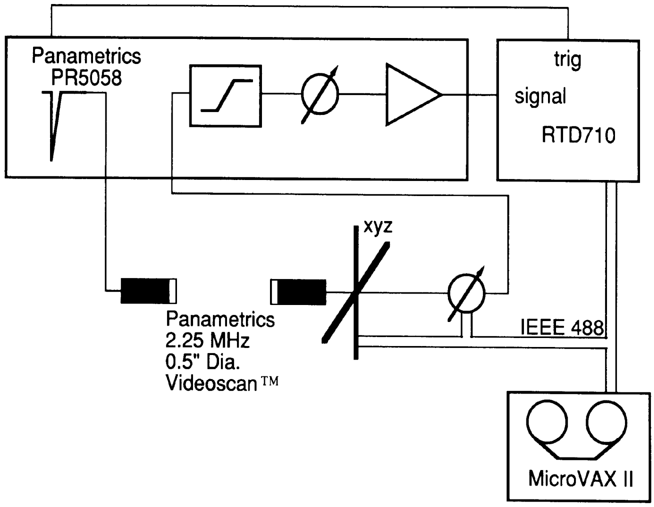

Fig. 2 Acquisition system for pulse-echo experiments.

was automatically gated to eliminate the multiples and any electromagnetic feedthrough from the drive signal at early time by finding the peak negative voltage, and retaining only the samples between 2.05 μs before the peak and 6.8 μs after the peak, ie, 180 samples. A mixed-radix FFT computed the Fourier transform (not used for Fig. 4) of the gated signal and the 21st spectral component, ie, at 2.222 MHz, retained for further use.

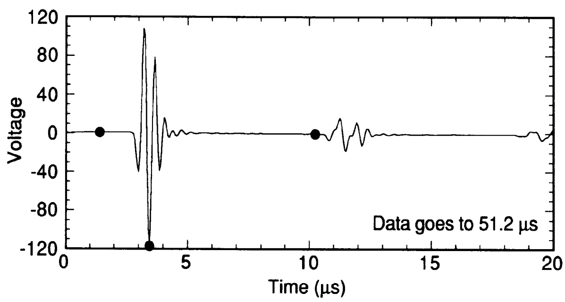

Fig. 3 Sample waveform collected by acquistion system. The black dots indicate the gate start, peak amplitude, and gate end, respectively.

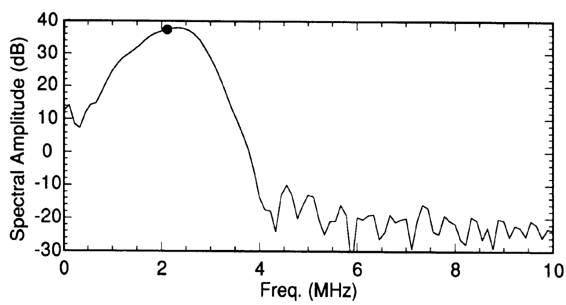

Fig. 4 Sample spectrum, that of Fig. 3. The dot indicates the frequency used for remainder of the paper.

Fig. 5 shows the magnitude of a set of data collected at an axial displacement of z=6 mm for radial displacements from r=0 mm to r=30 mm. The normalized axial distance $S=z\lambda/a^2=0.1$ indicates that this data is from the nearfield of the transducer. Eq. 3 was used to fit this (complex) data using Matlab's fmins nonlinear least-squares fitting algorithm [5] for N=15 beams. The starting points for the coefficients were those of Wen and Breazeale [2] for 15 beams. The error was radially weighted for minimization. The attenuation coefficient in the water was taken as 1.1 dB/mm, based on an independent measurement. The fit is seen to be very good, with an RMS error of 2.4%.

Fig. 6 shows the real and imaginary parts of the B_n and the magnitudes of the A_n plotted as error bars. The B_n correspond to the frequencies for a Fourier decomposition, and the A_n to the spectral values. In the Gaussian-beam decomposition, the B_n are not known in advance, but are computed as part of the optimization. It is gratifying that they are quite evenly spaced in the complex plane.

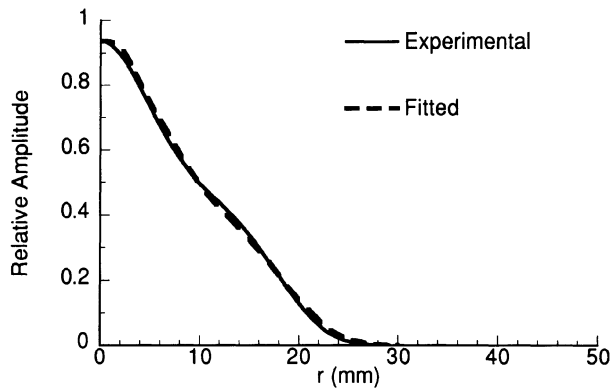

Fig. 5 Magnitude of 2.22 MHz data collected at z=6 mm

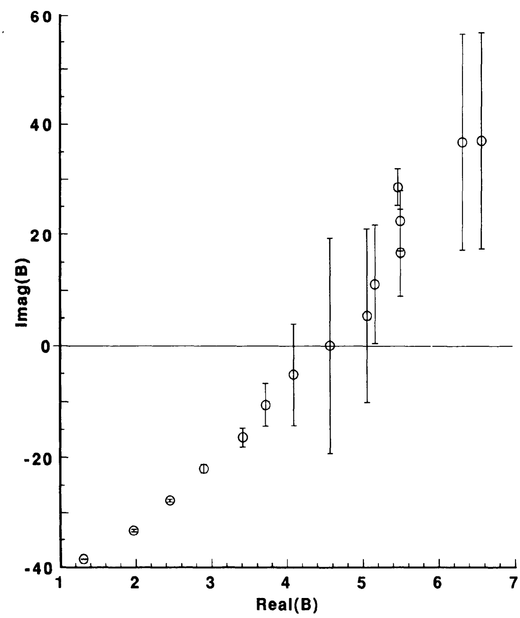

Fig. 6 B_n plotted on the complex plane. Bars indicate the magnitude of A_n.

Fig. 7 shows the normal surface velocity at the face of the transmitter which is consistent with the expansion found by fitting. It is clearly different than the uniform displacement assumed by Wen and Breazeale.

Figs. 8 and 9 show the measured and predicted pitch-catch responses at z=62 mm (S=1.0) and z=212 mm (S=3.55), ie, in the transition and far fields. For both distances the agreement is good, confirming that the fitted Gaussian-beam expansion is a good description of these transducers.

SUMMARY

We have derived an algebraic expression for the pitch-catch response of two identical transducers which are facing each other and have parallel axes, based on a description for each transducer that is a superposition of several, coaxial Gaussian beams. By perfoming a nonlinear, weighted least-squares optimization we have fitted the pulse-echo model (as opposed to a wave-field model) to a set of near-field pulse-echo data, and demonstrated that the fit is indeed good. Further, we have shown that based on this fit, the model can predict the pulse-echo response at other transducer displacements, over a wide range covering near field, far field, and transition zone.

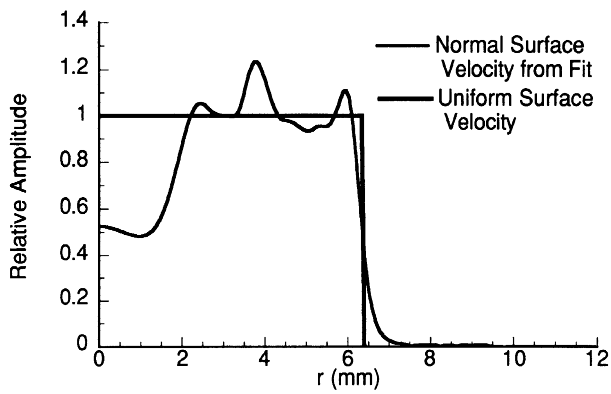

Fig. 7 Normal surface velocity at the face of the transmitter. For a real transducer, it is not expected to be uniform.

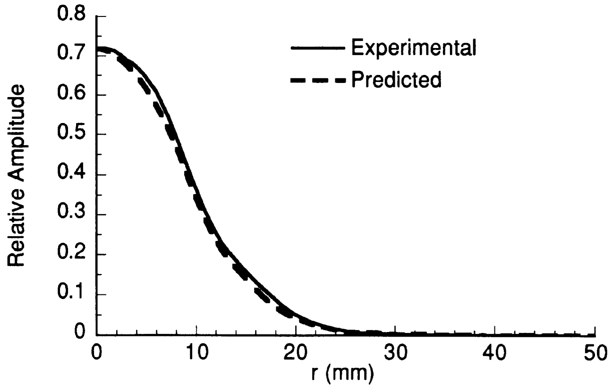

Fig. 8 Experimentally measured and Gaussian-Beam predictions of magnitude of pitch-catch response at z=62 mm.

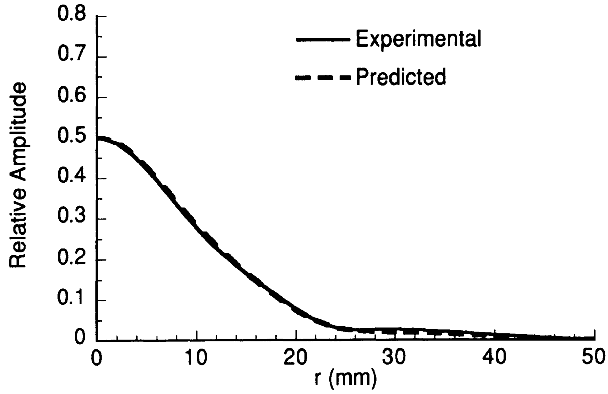

Fig. 9 Experimentally measured and Gaussian-Beam predictions of magnitude of pitch-catch response at z=212 mm.

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